# Magnetostatic modes in coupled antiferromagnet/ferromagnet bilayers

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**Abstract.** We study the influence of interface effects on the magnetostatic modes propagating in a coupled ferromagnetic/antiferromagnetic bilayer. We assume that the magnetic layers are thick enough to be described by the bulk parameters and they are coupled through the interaction between the magnetic moments located at the interface. We use a phenomenological approach taking into account the presence of different magnetic layers in the system to calculate the modified dynamical response of each material. We use the corrected magnetic permeability of the layers to obtain a correlation between the interface characteristics and the physical behavior of the magnetic excitations propagating in the system.

**PACS.** 75.40.Gb dynamic properties (dynamical susceptibility, static susceptibility, spin waves, spin diffusion, dynamic scaling, etc.) – 75.30.Cr (Saturation moments and magnetic susceptibilities) – 75.70.Cn Magnetic properties of interfaces (multilayers, superlattices, heterostructures) – 76.50.+g (Ferromagnetic, antiferromagnetic, and ferrimagnetic resonances; spin-wave resonance)

## **1** Introduction

For more than five decades it is known that ferromagnetic particles or films coated with antiferromagnetic materials exhibit a displaced hysteresis loop [1,2], but the origin of this shift, and some others related characteristics of these systems are still not very well understood. In the literature one can find different models to describe them, and the determination of the parameters used to represent the interactions is the key to have the best understanding of their physical characteristics. These parameters are also important to grow systems with some specific property. Therefore, it is quite desirable to find relations between these parameters and quantities that can be observed experimentally. With the knowledge of these relations, reliable numerical values for these quantities can be obtained by fitting experimental data. These quantities can be used to have a better comprehension of the physical properties of the system as well as to have information to develop materials to fit different purposes.

The behavior of magnetostatic modes propagating in a system constituted by non interacting magnetic layers is well known since long time ago [3,4]. In these systems, which usually have vacuum or a non magnetic spacer between the magnetic films, the bulk modes are weakly affected by the break of the symmetry and surface/interface

characteristics. On the other hand, the modes located at the surface and/or interfaces are sensitive to the physical characteristics of the surfaces and interfilm interactions [5], and from them one can extract information of the properties of the interface. It should be mentioned that some years ago Barnás and Grünberg [6] studied the behavior of spin waves propagating in layered systems composed by two interacting ferromagnetic films separated by a non-magnetic spacer. Recently, Liversey et al. [7] also studied spin waves in coupled magnetic media. Technological applications of multilayers systems have improved the interest on these systems and the analysis of their dynamical behavior is a very useful tool to investigate them. However, there are not many systematic studies or information on the correlation between the interface properties and the excitations in layered systems constituted by interacting films.

Among the layered systems that exhibit unusual physical behavior due to interaction between their constitutive materials, those constituted by an antiferromagnetic film grown directly on a ferromagnetic layer, under special conditions (temperature and dc magnetic field) are probably the most well studied [8] because the stability generated by the interfilm interaction may improve the quality of some devices [9,10].

Different theoretical models have been used to describe the characteristics of these systems [11–14] and the most remarkable difference is the description of the

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interface interactions. As mentioned above, the analysis of the modes localized at the interface certainly can give useful information on these properties. In this work we investigate the behavior of the magnetostatic modes propagating in systems constituted of an antiferromagnetic film grown in direct contact with a ferromagnetic layer. Numerical results are obtained considering that both films are described by their bulk parameters. A phenomenological approach is used to calculate the susceptibility of each layer with the appropriated corrections due to interface effects and the magnetic interaction with the neighbor layer. Special attention will be given to the influence of the unidirectional anisotropy (acting at the interface) and the interlayer interaction on the frequencies of the excitations. These quantities are responsible by the shift of the hysteresis loop as well as the stability of the magnetic system.

Our specific goal is to find a correlation between the interface properties and the frequency of the long wavelength excitation present in the system. To accomplish this purpose, we assume that the interface effects can be taken into account by considering that the contribution of the interface and the neighbor layer to the effective time independent field felt by all magnetic moments of a given medium, is the same of the magnetic moment at the interface. In other words, the first correction of the dynamical characteristics of the coupled system is the modification introduced in the effective field felt by its magnetic moments. Then the Landau-Lifshitz equation is used to obtain the susceptibility of each medium. This result allow us to calculate the dispersion relation of the magnetostatic modes propagating in the system. We change the parameters describing the interface effects to analyze their influence on the physical behavior of these modes. This approach should be useful to investigate magnetic bilayers with thicknesses much smaller than the wavelength of the magnetic excitations. A good discussion on the validity of this procedure can be found in a recent paper of Stamps and Usadel [15] and references therein.

### 2 Magnetostatic modes

In all calculation below we will use the following geometry: the uniaxial anisotropy of the antiferromagnetic film and the external static magnetic field are parallel to the surface of the layers, and both are in the z-direction; the y-direction is perpendicular to the surfaces and interface (see Fig. 1). We assume that the thicknesses  $d_3$  (antiferromagnetic) and  $d_2$  (ferromagnetic) of the layers allow us to describe their dynamical behavior by the bulk parameters with the corrections due to the presence of a magnetic layer nearby. These corrections are due to the interfilm interaction and the unidirectional anisotropy that, among other effects, is responsible for the shift of the hysteresis loop. We should remark that this description means that we are neglecting modifications of the spin distribution at the surface (spin reconstructions due to the symmetry break).

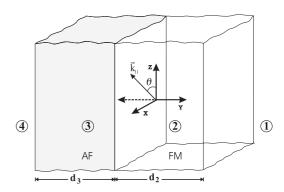


Fig. 1. Geometrical definitions of the axis mentioned in the text. The y-direction is perpendicular to the surfaces of the layers that are parallel to the *xz*-plane.

We name  $\vec{h}$  the field generated by the oscillations of the magnetization. Then, we use the Maxwell's equations to find that this field obeys  $\nabla \cdot (\mu^{(j)} \cdot \vec{h}) = 0$ , where j = 2and 3 indicates the ferromagnetic and antiferromagnetic medium respectively, and j = 1 and 4 denote the non magnetic medium where the system is embedded. Also, in the long wavelength limit (frequency region where the retarded effects are neglected) we have  $\nabla \times \vec{h} = 0$  and we may write  $\vec{h} = -\nabla \phi_m$ , where  $\phi_m$  obeys the equation:

$$\mu_{xx}^{(j)}\frac{\partial^2 \phi^{(j)}}{\partial x^2} + \mu_{yy}^{(j)}\frac{\partial^2 \phi^{(j)}}{\partial y^2} + \frac{\partial^2 \phi^{(j)}}{\partial z^2} = 0$$
(1)

where  $\phi_m$  is the scalar magnetic potential in the region j(with j as defined above). In presence of a static magnetic field  $\overrightarrow{H} = H_0 \widehat{z}$  the bulk magnetic permeability of each medium has the form:

$$\mu^{(j)} = \begin{pmatrix} \mu_{xx}^{(j)} & i\mu_{\perp}^{(j)} & 0\\ -i\mu_{\perp}^{(j)} & \mu_{yy}^{(j)} & 0\\ 0 & 0 & \mu_0 \end{pmatrix}.$$
 (2)

We notice that for a pure ferromagnetic layer (j = 2) we have the elements of the permeability tensor for given by [16]:

$$\mu_{xx}^{(2)} = \mu_{yy}^{(2)} = \frac{\omega_0^2 - \Omega^2 + 4\pi\omega_S^F\omega_0}{\omega_0^2 - \Omega^2}$$
(3)

and

$$\mu_{\perp}^{(2)} = \frac{4\pi\omega_S^F \Omega}{\omega_0^2 - \Omega^2} \tag{4}$$

where  $\omega_0 = \gamma H_0$ , and  $\omega_S^F = \gamma M_S^F$  ( $\gamma$  is the gyromagnetic factor and  $M_S^F$  the saturation magnetization of the ferromagnet). On the other hand, the permeability of a pure antiferromagnetic medium with the external dc field parallel to the uniaxial anisotropy has the elements of the tensor given by [17]:

$$\mu_{xx}^{(3)} = \mu_{yy}^{(3)} = 1 + 4\pi\omega_S^{AF}\omega_A \left[f_+(\Omega) + f_-(\Omega)\right], \quad (5)$$

$$\mu_{\perp}^{(3)} = -4\pi\omega_S^{AF}\omega_A \left[f_+(\Omega) - f_-(\Omega)\right],\tag{6}$$

where

$$f_{\pm}(\Omega) = \frac{1}{\Omega_0^2 - (\Omega \pm \omega_0)^2}.$$
 (7)

In the equations above  $\omega_S^{AF} = \gamma M_S^{AF}$ ,  $\Omega_0^2 = \gamma^2 (2H_E H_A + H_A^2)$  and  $\omega_A = \gamma H_A$ , where  $M_S^{AF}$  is the saturation magnetization of the sublattice of the antiferromagnet,  $H_E$  and  $H_A$  are the exchange and anisotropy fields, respectively, and  $\Omega_0$  is the zero-field antiferromagnetic resonance frequency. The identity tensor describes the magnetic permeability of the non-magnetic medium where the system is embedded.

We write the solutions for equation (1) as  $\phi_m = e^{i(\vec{k}_{||}\cdot\vec{r} - \Omega t)}\phi$  with  $\phi$  given by:

$$\phi = \begin{cases} A_1 e^{[-k_{||}(y-d_2)]}, & y \ge d_2\\ A_{21} e^{k_y^{(2)}y} + A_{22} e^{-k_y^{(2)}y}, & 0 \le y \le d_2\\ A_{31} e^{k_y^{(3)}y} + A_{32} e^{-k_y^{(3)}y}, & -d_3 \le y \le 0\\ A_4 e^{[k_{||}(y+d_3)]}, & y \le -d_3 \end{cases}$$
(8)

where  $k_{||}$  is the component parallel to the surface of the wave vector  $\vec{k}$ . The y component of the wave vector is written as a function of the angle  $\theta$  between  $\vec{k}_{||}$  and the z direction as:

$$\left[\frac{k_y^{(j)}}{k_{||}}\right]^2 = -\left(\frac{\mu_{xx}^{(j)}sin^2(\theta) + cos^2(\theta)}{\mu_{yy}^{(j)}}\right),\tag{9}$$

The dispersion relation is obtained from the homogeneous system of equations resulting of the requirement that the fields must obey the Maxwell boundary conditions at the interfaces and surfaces. As all equations above have an explicit dependence on magnetic permeability, the frequencies of the magnetostatic modes should also be dependent on this quantity, and then, they should have information about all interaction that modify the dynamical response of the system. Our task is to include these modifications in the magnetic susceptibility.

## 3 The modified susceptibility

We consider that the interaction between the layers has the energy given by  $\frac{-H_I}{\langle M \rangle} \vec{m}_2 \cdot \vec{m}_3$ , where  $\vec{m}_2$  and  $\vec{m}_3$  are the values of the magnetization at the surfaces of the interface, and  $\langle M \rangle$  is the averaged value of the saturation magnetization of the two media. Therefore, the effective field felt by the magnetic moments located at the medium j depends on the magnetic moments located at the neighbor medium j' ( $j \neq j'$ ) and  $H_I$  is the parameter that measures the energy necessary to rotate 180 degree one of the magnetization with respect to the other. Therefore, it should be expected that the frequencies of the modes have a direct dependence on  $H_I$ .

The presence of a magnetic layer in the neighborhood has two effects: first, the static contribution modifies the magnetic permeability tensor of the correspondent film (it should represent the tendency of the higher magnetization to stabilize the lower one) and second it contributes to the time dependent value of the magnetic induction. On the other hand, the unidirectional anisotropy  $(H_{ad})$  contributes to stabilize the magnetic moments at both layers and also modifies the effective magnetic permeability tensor because it changes the effective field felt by the magnetic moments.

We assume that the contribution of the interface and neighbor magnetic layer to the effective field felt by all magnetic moments at one medium is the same one felt by the magnetic moments localized at its side of the interface. This approach is similar to one recently used by Stamps and Usadel [15] to study dynamical characteristics of these systems. In fact, these authors suggest that this procedure can be used in the Landau-Lifshitz-Gilbert equations under the assumption of slow dynamics (long wavelength limit). This is what is done in this paper and to materialize it we consider that the ferromagnetic material is one like a Co or Fe, that have their magnetic permeability tensor given by the general form showed above. A simple calculation allow us to obtain that the correction of their elements is obtained through the substitution of the static field  $H_0$  by  $H_0 + \frac{H_I m_3^0}{\langle M \rangle} + H_{ad}$ , where  $m_3^0$  is the saturation magnetization of the antiferromagnetic film. An analogous procedure (a little more cumbersome) gives a similar result for the static correction of the magnetic permeability tensor of the antiferromagnetic material. We consider that the unidirectional anisotropy is also parallel to the z direction to find that the correction of permeability tensor of the antiferromagnetic medium corresponds to the substitution of  $\omega_0 = \gamma H_0$  by  $\gamma \left( H_0 + \frac{H_I m_0^2}{\langle M \rangle} + H_{ad} \right)$ , where  $m_2^0$ is the saturation magnetization of the ferromagnetic film. The validity of this approach is discussed in [15].

These results allow us to calculate the effective magnetic and induction fields generated by the oscillations of the magnetic moments immediately above ( $\varepsilon_+$ ) and bellow ( $\varepsilon_-$ ) the interface. They are given by:

$$\vec{h}(\varepsilon_{+}) = -\nabla\phi^{(2)} - \frac{H_{I}}{4\pi\langle M \rangle} \times (\tilde{\mu}^{(3)} - I) \cdot \nabla\phi^{(3)}, \qquad (10)$$

$$\times \overrightarrow{b}(\varepsilon_{+}) = -\widetilde{\mu}^{(2)} \cdot \nabla \phi^{(2)} - \frac{H_{I}}{4\pi \langle M \rangle}$$
$$\times \widetilde{\mu}^{(2)} \cdot (\widetilde{\mu}^{(3)} - I) \cdot \nabla \phi^{(3)} \tag{11}$$

$$\overrightarrow{h}(\varepsilon_{-}) = -\nabla\phi^{(3)} - \frac{H_I}{4\pi\langle M \rangle}$$

$$\times (\tilde{\mu}^{(2)} - I) \cdot \nabla \phi^{(2)}, \tag{12}$$

$$\overline{b}(\varepsilon_{-}) = -\widetilde{\mu}^{(3)} \cdot \nabla \phi^{(3)} - \frac{\Pi_{I}}{4\pi \langle M \rangle} \\
\times \widetilde{\mu}^{(3)} \cdot (\widetilde{\mu}^{(2)} - I) \cdot \nabla \phi^{(2)}.$$
(13)

where  $\tilde{\mu}^{(j)}$  is the modified permeability of the medium j.

In order to relate the interface effects with the properties of the magnetic excitations, we follow the standard procedure to obtain the dispersion relation for these modes. Then, the continuity of the parallel components of  $\overrightarrow{h}$  and the perpendicular component of the induction field  $\overrightarrow{b}$  at the interface  $[h_{||}(\varepsilon_{+}) = h_{||}(\varepsilon_{-})$  and  $b_{\perp}(\varepsilon_{+}) = b_{\perp}(\varepsilon_{-})$ ] allows us to construct a system of equations which, from the condition for a nontrivial solution, we find that the implicit dispersion relation of the magnetostatic modes in these system is given by:

$$\left\{ad^{*}e^{-iK_{y}^{(2)}k_{||}d_{2}} - a^{*}de^{iK_{y}^{(2)}k_{||}d_{2}}\right\} \times \left\{be^{-iK_{y}^{(3)}k_{||}d_{3}} - b^{*}e^{iK_{y}^{(3)}k_{||}d_{3}}\right\} + \left\{bc^{*}e^{-iK_{y}^{(3)}k_{||}d_{3}} - b^{*}ce^{iK_{y}^{(3)}k_{||}d_{3}}\right\} \times \left\{ae^{-iK_{y}^{(2)}k_{||}d_{2}} - a^{*}e^{iK_{y}^{(2)}k_{||}d_{2}}\right\} = 0, \quad (14)$$

with

$$\left[K_y^{(j)}\right]^2 = -\left(\frac{\widetilde{\mu}_{xx}^{(j)}sin^2(\theta) + cos^2(\theta)}{\widetilde{\mu}_{xx}^{(j)}}\right).$$
 (15)

and

$$\begin{split} a &= iK_{y}^{(2)}\widetilde{\mu}_{xx}^{(2)} - \sin(\theta)\widetilde{\mu}_{\perp}^{(2)} - 1.\\ b &= iK_{y}^{(3)}\widetilde{\mu}_{xx}^{(3)} + \sin(\theta)\widetilde{\mu}_{\perp}^{(3)} - 1.\\ c &= iK_{y}^{(3)}\widetilde{\mu}_{xx}^{(3)} + \sin(\theta)\widetilde{\mu}_{\perp}^{(3)}\\ &-\mathcal{H}_{\mathcal{I}}\left\{ik_{y}^{(3)}\left[\widetilde{\mu}_{xx}^{(2)}(\widetilde{\mu}_{xx}^{(3)} - 1) + \widetilde{\mu}_{\perp}^{(2)}\widetilde{\mu}_{\perp}^{(3)}\right]\right\}\\ &-\mathcal{H}_{\mathcal{I}}\left\{\sin(\theta)\left[\widetilde{\mu}_{xx}^{(2)}\widetilde{\mu}_{\perp}^{(3)} + \widetilde{\mu}_{\perp}^{(2)}(\widetilde{\mu}_{xx}^{(3)} - 1)\right]\right\}.\\ d &= ik_{y}^{(2)}\widetilde{\mu}_{xx}^{(2)} - \sin(\theta)\widetilde{\mu}_{\perp}^{(2)}\\ &-\mathcal{H}_{\mathcal{I}}\left\{ik_{y}^{(2)}\left[\widetilde{\mu}_{xx}^{(3)}(\widetilde{\mu}_{xx}^{(2)} - 1) + \widetilde{\mu}_{\perp}^{(2)}\widetilde{\mu}_{\perp}^{(3)}\right]\right\}\\ &+\mathcal{H}_{\mathcal{I}}\left\{\sin(\theta)\left[\widetilde{\mu}_{xx}^{(3)}\widetilde{\mu}_{\perp}^{(2)} + \widetilde{\mu}_{\perp}^{(3)}(\widetilde{\mu}_{xx}^{(2)} - 1)\right]\right\}\end{split}$$

where  $\mathcal{H}_{\mathcal{I}} = \frac{H_I}{4\pi \sqrt{M_S^F M_S^{AF}}}$ . A special attention should be given to the terms of equation (14) that are proportional to  $H_I$ . From it comes the main contribution for the modification introduced in the frequency of the modes from the interaction of the films.

#### Numerical results 4

We use  $H_E = 540$  kG,  $H_A = 200$  kG,  $M_S^{AF} = 0.56$  kG and  $M_S^F = 1.6$  kG, which are physical parameters that describe the  $Fe/FeF_2$  bilayer, to obtain numerical results for the frequency of magnetostatic modes propagating in these systems as a function of the direction of the propagation, for different sets of the interface parameters.

The dispersion relation for a non interaction bilayer is depicted in Figure 2 for  $k_{\parallel}d_2 = 1$ ;  $k_{\parallel}d_3 = 4$ . Its comparison with the results for interacting systems allow us to see the influence of the interface effects on the optical behavior of the system. We remark that the complete calculation of the potentials (or fields) shows that the lowest frequency

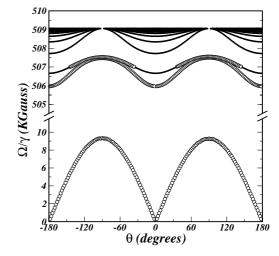


Fig. 2. Dispersion relation for magnetostatic modes propagating parallel to the surface for different values of the angle between the wavevector and the z-direction for the case where there is no anisotropy and interaction between the constitutive films. The open circles are the localized modes, and the black circles are the confined modes. The parameters used are:  $H_0 = H_I = H_{ad} = 0; \ k_{\parallel} d_2 = 1; \ k_{\parallel} d_3 = 4.$ 

branches are modes located at the interface. They should be sensitive to the interface effects.

The dispersion relation depicted in Figure 3 was obtained for the same system studied in Figure 2 with the addition of an unidirectional anisotropy of 500 kG. This result shows that the main consequence of this anisotropy is to create new branches of confined modes that propagate in a finite angular region. The effect of the unidirectional anisotropy is almost the same of an externally applied dc magnetic field. The size of the angular region depends on the intensity of the anisotropy. The intensity of the unidirectional anisotropy determines the size of the angular region around the anisotropy direction where these modes can propagate. We show in Figure 4 the limit angle as a function of the intensity of the unidirectional anisotropy.

In Figure 5 we plot the dispersion relation for the same system, but now we consider that the interfilm interaction has a finite value. From this result we may say that the main effect of this interaction is to create asymmetries in the dispersion relation. To be more precise we should say that the modes located at the interface have their dispersion relation affected qualitative and quantitatively by  $H_I$ . For example, a plot of the intensity of the magnetic field shows that the lowest frequency branches corresponds to modes located at the interface. These modes perceive the strong anisotropy of the antiferromagnetic material via the interfilm interaction and the result is the non reciprocal behavior depicted in the figure. Similar effect is also observed for the modes with frequency near of the antiferromagnetic resonance. These modes are strongly dependent on the characteristics of the antiferromagnetic medium, but the presence of the ferromagnetic layer produce a extra field that introduces an additional anisotropy which

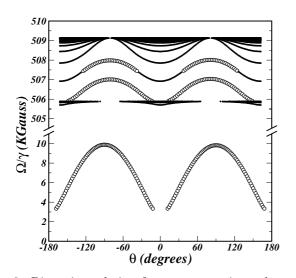


Fig. 3. Dispersion relation for magnetostatic modes propagating parallel to the surface for different values of the angle between the wavevector and the anisotropy direction (z-direction) for the case where there is no interaction between the constitutive films but the unidirectional anisotropy is 500 G. The symbols and the parameters are the same as in Figure 2.

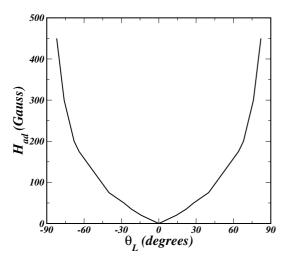


Fig. 4. Limit angle of the intermediate branch created by the unidirectional anisotropy as a function of its intensity  $H_{ad}$ .

is "transmitted" to this medium via the contact interaction  $H_I$ . As a result, for  $\theta > 0$  and  $\Omega/\gamma$  between 506 and 507 kGauss, the lower frequency branch, which corresponds to a mode located at the interface, exist only into a finite angular region. On the other hand the modes propagating in the opposite direction are located at the free surface of the antiferromagnet and has a quite regular behavior. The branch observed for  $\theta > 0$  and  $\Omega/\gamma$  close to 507 kGauss corresponds to a mode located at the free surface and it is weakly affected by the presence of a different material at the neighborhood. In summary, this interaction is responsible for the introduction of a non-reciprocal behavior in the sense that modes located in the interface are affected by the presence of different materials.

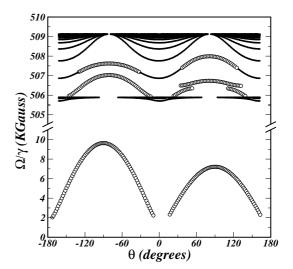


Fig. 5. Dispersion relation for magnetostatic modes propagating parallel to the surface for different values of the angle between the wavevector and the z-direction for the case where there is no unidirectional anisotropy but the interfilm interaction has a finite value. The symbols and the parameters are the same as in Figure 2. However, here we have  $H_I = 300$  G.

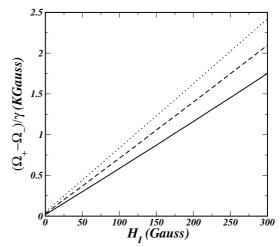


Fig. 6. Difference between the frequencies of the lowest frequency modes propagating in the system for different directions of propagation. Solid line for  $\theta = 45^{\circ}$ , dashed line for an angle  $\theta = 60^{\circ}$ , and dotted line for  $\theta = 90^{\circ}$ .

The difference between the frequencies of the lowest frequency modes propagating in opposite directions as a function of the intensity of the interfilm interaction for different directions of propagation is shown in Figure 6. It can be observed that the non-reciprocity depends on the direction considered, but in all situations analyzed we found a linear relation between the intensity of the interfilm interaction and the difference of the frequencies.

## 5 Comments and conclusions

We have calculated the dispersion relation of the magnetostatic modes propagating in a bilayer constituted of a antiferromagnetic layer growing in direct contact with an ferromagnetic substract. We consider that there is an unidirectional anisotropy at the interface and the layers are coupled. In our calculation we take into account this fact through the calculation of the correction on the magnetic permeability of each medium. With the corrected magnetic permeability we analyzed separately the unidirectional anisotropy and the interfilm interaction. We could see that while the effect of the unidirectional anisotropy is almost the same of an external dc magnetic field (only the appearance of new branches of confined modes propagating in a finite region of the space [17]), the interfilm interaction introduces an additional effect on the character of selected localized modes: the non reciprocal behavior. Moreover, we also could see that there is a linear dependence between the difference in the frequencies of the modes propagating in opposite directions and the intensity of the interfilm interaction. It should be remarked that the interface effects are the result of the thermal treatment of the sample and they depend on the thermal history of the sample. Our results show that the analysis of magnetostatic modes can be used to characterize these system and have valuable information on the physical characteristics of the interface.

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